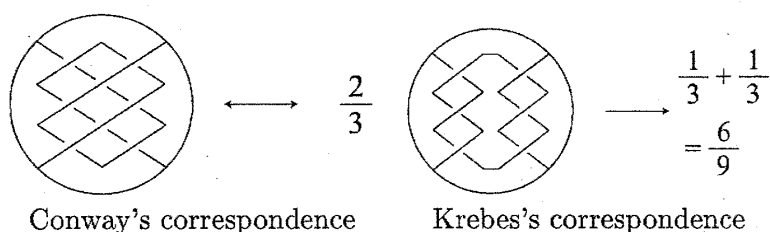


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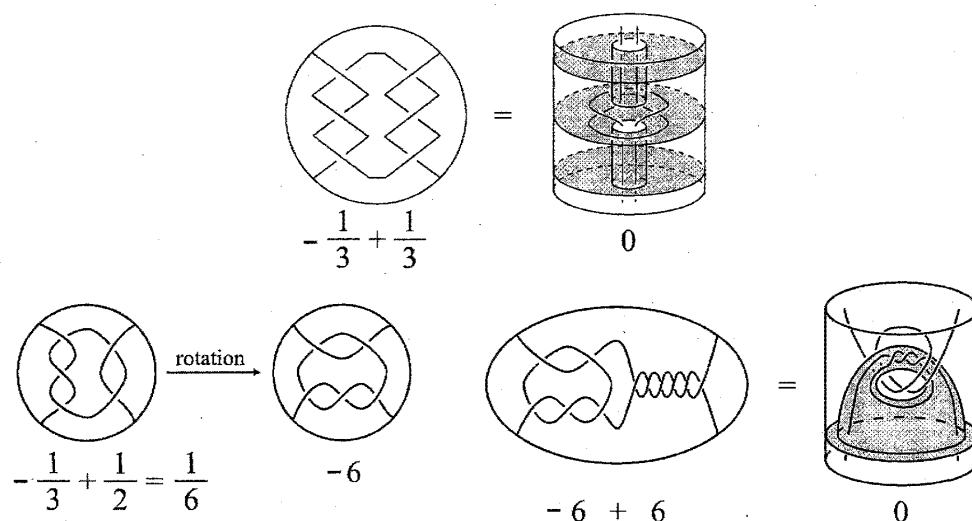
Rational structure on algebraic tangles and closed incompressible surfaces in the complements of algebraically alternating knots and links

Makoto Ozawa
Komazawa University

In 1970, Conway ([1]) showed that the rational tangles correspond to the rational numbers in one-to-one. In 1999, Krebs ([2]) constructed a map from tangles to formal fractions (not necessarily reduced), and the map on the algebraic tangles is surjective.

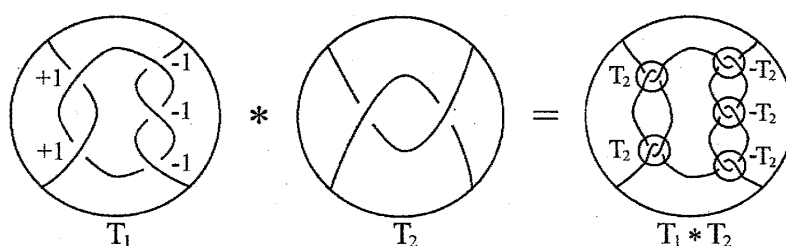


Theorem 1. *Let (B, T) be an algebraic tangle, $F \subset B - T$ an essential surface. Then, F separates the components of T in B . Moreover, $B - T$ contains at least one essential surface, and the boundary slope of essential surfaces are unique.*



By Theorem 1, a map ϕ from algebraic tangles to the boundary slopes of essential surfaces is defined.

We define the *multiplication* $T_1 * T_2$ of two tangles T_1 and T_2 like a figure.

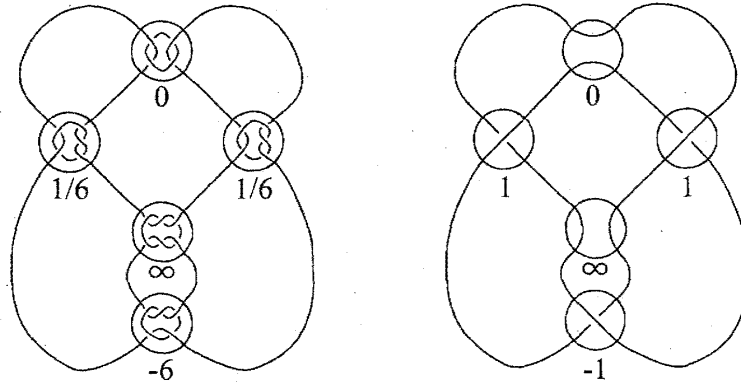


Theorem 2. *The map ϕ is a homomorphism from algebraic tangles to rational numbers. Namely, the following hold.*

- $\phi(T_1 + T_2) = \phi(T_1) + \phi(T_2)$
- $\phi(T_1 * T_2) = \phi(T_1)\phi(T_2)$
- $\phi(-T) = -\phi(T)$
- $\phi(T^*) = -\frac{1}{\phi(T)}$

Here, $+$ denotes the tangle sum, $*$ the tangle multiplication, $-$ the reflection and $*$ the rotation.

In Conway notation \tilde{K} , we replace each algebraic tangle T with a rational tangle of slope $+1$ (resp. -1 , 0 , ∞) if $\phi(T) > 0$ (resp. < 0 , $= 0$, $= \infty$), and the resultant diagram \tilde{K}_0 is called the *basic diagram* of \tilde{K} . We say that \tilde{K} is *algebraically alternating* if \tilde{K}_0 is alternating, and that K is *algebraically alternating* if K has an algebraically alternating diagram. The class of algebraically alternating links contains both of algebraic links and alternating links.



\tilde{K} : algebraically alternating diagram \tilde{K}_0 : basic diagram of \tilde{K}

Theorem 3. *Let (S^3, K) be an algebraically alternating links, $F \subset S^3 - K$ an essential closed surface. Then, F separates the components of K in S^3 . Moreover, the basic diagram \tilde{K}_0 is split, or F is contained in an algebraic tangle of (S^3, K) . In particular, if F is a 2-sphere, then there exists a cut tangle. If F is a torus and there exists no cut tangle, then (S^3, K) contains Q_2 .*

Corollary. *Any essential closed surface in the complement of an algebraically alternating knot is meridionally compressible.*

References

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